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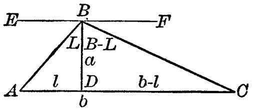
For the plane triangle, let ABC be any plane triangle. BD is a $\perp$ to AC from B. The vertex B may move anywhere on the straight line EF which is parallel to AC, thus keeping the altitude BD constant in value. The angle B is broken up into the two parts L and $(B - L)$ and the corresponding opposite sides are l and $(b - l)$.



In the $\triangle ABD$, $\tan L = l/a$ and in the $\triangle BDC$,

$$\tan (B - L) = \frac{b - l}{a}.$$

Now

$$B = L + (B - L) = \tan^{-1} (l/a) + \tan^{-1} \frac{b - l}{a}.$$

Differentiating,

$$\frac{dB}{dl} = a \left[\frac{1}{a^2 + l^2} - \frac{1}{a^2 + (b - l)^2} \right]$$

which is 0 if $b = 2l$, giving a maximum value of B. But if $b = 2l$, $AB = BC$ and the triangle is isosceles.

Also solved by M. R. Gaffet.

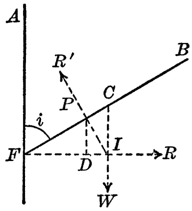
MECHANICS.

326. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

A uniform beam, of length $2l$, rests in equilibrium against a smooth vertical wall and upon a peg at a distance a from the wall, show that the inclination of the beam to the vertical is

$$\sin^{-1} \left(\frac{a}{l} \right)^{\frac{1}{3}}.$$

Solution by H. S. Uhler, Yale University.



In order to obtain the given inclination it is necessary to assume that the reaction of the peg is normal to the beam, for, if this be not the case, the beam may be in equilibrium on various slants depending upon the value of the coefficient of friction.

The necessary and sufficient condition for static equilibrium is that the lines of action of the three forces acting on the beam pass through a single point, I. These forces are obviously the weight of the beam, W, acting vertically downward, the horizontal reaction of the *smooth* wall, R, and the normal reaction of the peg, R'. $\overline{BC} = \overline{CF} = l$. $\overline{DF} = a$. $P \equiv$ peg. Let $\overline{FP} \equiv m$ and $\overline{FI} \equiv n$. Since $\overline{IP}$ and $\overline{PD}$ are, respectively, the perpendiculars dropped from the vertices of the right angles upon the hypotenuses of the triangles CIF and FPI we have $n^2 = lm$ and $m^2 = an$, so that $a^3/m^3 = a/l$ or $a/m = (a/l)^{1/3}$. But $i = \sin^{-1} (a/m) = \sin^{-1} (a/l)^{1/3}$, as required.

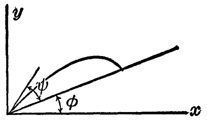
Also solved by A. M. Harding, L. A. Warren, G. Paaswell, Paul Capron, O. S. Adams, and Horace Olson.

327. Proposed by C. N. SCHMALL, New York City.

An inclined plane makes an angle φ with the horizontal plane, and from its foot a body is projected upward at an angle ψ to the plane, and with a velocity v. Show that it will strike the plane perpendicularly if $\tan \psi = \frac{1}{2} \cot \varphi$ and that its range up the plane in this case will be

$$\frac{2v^2 \sin \varphi}{g(1 + 3 \sin^2 \varphi)}.$$

SOLUTION BY A. M. HARDING, University of Arkansas.



The position of the projectile at any time is given by

$$x = v \cos(\varphi + \psi)\cdot t,$$

$$y = v \sin(\varphi + \psi)\cdot t - \tfrac{1}{2}gt^2.$$

Eliminating t we obtain, as the equation of the path,

$$y = x \tan(\varphi + \psi) - \frac{gx^2}{2v^2 \cos^2(\varphi + \psi)}. \tag{1}$$

Let r be the range up the plane. Then the coördinates of the point where the projectile meets the inclined plane are

$$x = r \cos\varphi, \qquad y = r \sin\varphi. \tag{2}$$

Substituting these in (1), we find

$$r = \frac{2v^2}{g}\cdot\frac{\cos(\varphi + \psi)\sin\psi}{\cos^2\varphi}. \tag{3}$$

From (1) we find

$$\frac{dy}{dx} = \tan(\varphi + \psi) - \frac{gt}{v\cos(\varphi + \psi)}. \tag{4}$$

This gives the direction at any time t. The projectile strikes the plane after a time

$$t = \frac{r\cos\varphi}{v\cos(\varphi + \psi)} = \frac{2v\sin\psi}{g\cos\varphi}.$$

Hence, when the projectile strikes the plane, we have

$$\frac{dy}{dx} = \tan(\varphi + \psi) - \frac{2\sin\psi}{\cos\varphi\cos(\varphi + \psi)}$$

$$= \frac{\sin(\varphi + \psi)\cos\varphi - 2\sin\psi}{\cos\varphi\cos(\varphi + \psi)}.$$

If the projectile strikes perpendicularly, $dy/dx = -\cot\varphi$. Hence,

$$\frac{\sin(\varphi + \psi)\cos\varphi - 2\sin\psi}{\cos\varphi\cos(\varphi + \psi)} = -\frac{\cos\varphi}{\sin\varphi}.$$

From this equation, we find $\tan\psi = \tfrac{1}{2}\cot\varphi$. Substituting in (3), we find

$$r = \frac{2v^2\sin\varphi}{g(1 + 3\sin^2\varphi)}.$$

Also solved by H. C. FEEMSTER, H. S. UHLER, G. PAASWELL, C. N. SCHMALL, O. S. ADAMS, HORACE OLSON, and L. A. WARREN.

NUMBER THEORY.

242. Proposed by NORMAN ANNING, Chilliwack, B. C.

Find a function of n which is equal to A_k when $n \equiv k \pmod{p}$, $k = 1, 2, 3, \cdots, p$.

SOLUTION BY THE PROPOSER.

Let θ be a primitive root of $x^p - 1 = 0$, then it is known that $\theta, \theta^2, \theta^3, \cdots, \theta^p$ are all the pth roots of unity and that $\theta^m + \theta^{2m} + \theta^{3m} + \cdots + \theta^{pm} = p$ or 0, according as m is or is not divisible by p. [See, for example, Burnside and Panton, *Theory of Equations*, Vol. I, pp. 95 and 96.] Consider now the expression:

$$\begin{aligned} f = \frac{1}{p}\,[&A_1\{\theta^{n-1} + \theta^{2(n-1)} + \cdots + \theta^{p(n-1)}\} \\ &+ A_2\{\theta^{n-2} + \theta^{2(n-2)} + \cdots + \theta^{p(n-2)}\} \\ &+ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ &+ A_p\{\theta^{n-p} + \theta^{2(n-p)} + \cdots + \theta^{p(n-p)}\}]. \end{aligned}$$